Addendum to the Weak Parity-Violating Pion-Nucleon Coupling

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We need to make a correction to our earlier work [1], namely a critical error in the relative sign between the lowest-dimensional term and the leading nonperturbative contribution. By correcting the sign error and evaluating an additional contribution which we omitted earlier, we conclude that our new prediction on the weak pion-nucleon coupling $f_{\pi NN}$ is in the vincinity of 3×10^{-7}, and is stable.

The main idea of our approach is that both the strong and parity-violating pion-nucleon couplings, $g_{\pi NN}$ and $f_{\pi NN}$, respectively, are given by the lowest dimensional diagram and by a pion vacuum susceptibility, χ_π. Therefore, by using the known value of $g_{\pi NN}$ one can determine χ_π and thereby predict the value of $f_{\pi NN}$. Using definitions for the susceptibilities given in Ref.[1], the QCD sum rule for the strong πNN coupling

is[2]

$$M_B^6 L^{-4/9} E_2 - \chi_\pi a M_B^4 L^{2/9} E_1 - \tfrac{11}{24} < g_c^2 G^2 > M_B^2 E_0 + \tfrac{4}{3} a^2 L^{4/9} + \tfrac{1}{3} m_0^2 a^2 \frac{L^{-2/27}}{M_B^2}$$

$$= \{g_{\pi NN} + B M_B^2\} \bar{\lambda}_N^2 \, e^{-M^2/M_B^2}. \tag{1}$$

This equation is the same as Eq. (16) in Ref.[1] except for changes in coefficients of terms whose contributions are negligible within the errors of the method, an addition of a dimension eight term, which also does not alter our results, and an explicit inclusion of a monpole term from the continuum. As was discussed in detail in Ref.[1], from the dominant first two terms on the left hand side of the above sum rule and by making use of the known value of $g_{\pi NN}$ we find the value of $\chi_\pi a$ is about -1.9 GeV^2; within the errors of both calculations, this value is close to that obtained in a three-point evaluation[3] using nonlocal condensates. It is notable that we have shown explicitly that the large value of the strong πNN coupling requires nonperturbative QCD.

In a subsequent sum rule calculation in which the chiral-quark-model value for the π-quark coupling constant, $g_{\pi q}$, and a value for $\chi_\pi a$ in the vicinity of $-1.5\,GeV^2$ were used[2], a value of $g_{\pi NN} = -(14.8 \pm 0.7)$ for $M_B = (1.10 \pm 0.05)\,GeV$ was obtained. This is a fairly stable result with respect to the Borel mass. An additional error of at least ± 2.0 should be understood in view of the uncertainties involved in the various condensates.

We turn our attention to the weak p.v. pion-nucleon coupling, which is defined as follows [4]

$$\mathcal{L}_{\pi NN}^{p.v.} = \frac{f_{\pi NN}}{\sqrt{2}} \bar{\psi}(\vec{\tau} \times \vec{\pi})_3 \psi. \tag{2}$$

Only charged pions can be emitted or absorbed. Here we modify the QCD sum rule which we obtained earlier[1] (and which has some pathological behavior as the Borel mass increases) by multiplying both sides (LHS and RHS) by the factor $(p^2 - M^2)$ immediately before taking the Borel transform. This has helped to produce a QCD sum rule which is very well behaved.

$$\frac{G_F \sin^2 \theta_W (\tfrac{17}{3} - \gamma)}{24\pi^2} \, [(4 M_B^{10} - M^2 M_B^8) L^{-4/9} E_3$$

$$
\begin{aligned}
- \quad & 4(M_B^8 - \frac{1}{3}M^2 M_B^6)\chi_\pi \, aL^{-4/9}E_2 - (M_B^6 - \frac{1}{2}M^2 M_B^4)m_0^\pi \, a \, E_1 L^{-4/9}] \\
= \quad & \{f_{\pi NN} + B'M_B^2\}\bar{\lambda}_N^2 \, 2M^2 \, e^{-M^2/M_B^2}. \tag{3}
\end{aligned}
$$

This sum rule still suffers from the fact that the contribution from the gluon condensate diagrams is yet to be included; these diagrams are much more complicated to evaluate although they are of the same order or smaller than uncertainties of our calculation.

Numerically, we obtain $f_{\pi NN} = (3.04 \pm 0.01) \times 10^{-7}$ for $M_B = (1.10 \pm 0.05)\,GeV$, a prediction which is even more stable than the other sum rules (on the nucleon mass and the strong pion-nucleon coupling). The uncertainties in the condensates and in the terms which have been neglected amount to at least $\pm 0.5 \times 10^{-7}$. About 50 % of the contribution to $f_{\pi NN}$ comes from the nonperturbative $\chi_\pi a$ term. The present prediction on $f_{\pi NN}$ is in much better agreement with that of Desplanques, Donoghue and Holstein [4] and also with that obtained independently by one of us [2].

In summary, our published Letter [1] unfortunately contains a sign error which completely changes the results. In particular, the sign in front of the $(2/3)\chi_\pi aL^{-4/9}M_B^2 E_2$ in Eq. (14) should be $-$ and not $+$. The first (lowest-dimensional) and second (nonperturbative) terms then add rather than subtract.

We have also evaluated two diagrams that we omitted in our earlier work, namely Figs. 3d and 3e, which do appear in the external field method, because it is an internal pion which couples to the gauge boson. However, we find that these diagrams make a negligible contribution.

References

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